Exam 2: Ch 3 & 4 NAME Answer Key

Math 176, Precalculus, Section 6265

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100 points. Show all work to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!

1. (16 pts) Consider the polynomial $P(x) = x^4 - 2x^3 + 5x^2 - 8x + 4$.

a) List all possible rational zeros of *P(x)*.

2 $\pm 1, \pm 2, \pm 4$

b) Find all rational and complex zeros of *P(x)*. Use the quadratic formula if necessary.

8

$$\begin{array}{r|rrrrr} 1 & 1 & -2 & 5 & -8 & 4 \\ & 0 & 1 & -1 & 4 & -4 \\ \hline & 1 & -1 & 4 & -4 & 0 \end{array}$$

$x^3 - x^2 + 4x - 4 = 0$

$x^2(x-1) + 4(x-1) = 0$

$(x-1)(x^2+4)$

$x^2 = -4$

$x = \pm 2i$

RATIONAL ZEROS

1, MULT OF 2

COMPLEX:

$\pm 2i$

c) Write the complete factorization of *P(x)*.

2 $P(x) = (x-1)^2(x-2i)(x+2i)$

d) What is the remainder when *P(x)* is divided by $x+3$. What is its significance?

$$\begin{array}{r|rrrrr} -3 & 1 & -2 & 5 & -8 & 4 \\ & 0 & -3 & 15 & -60 & 204 \\ \hline & 1 & -5 & 20 & -68 & 208 \end{array}$$

4

$P(-3) = 208$

REMAINDER = 208

MP = -16 ADD TO -6

2. (22 pts) Consider the rational function $R(x)=\frac{2x^2-6x-7}{x^2-2x-8}$. $= \frac{2x^2-6x-7}{(x-4)(x+2)}$

6

a) Find the x- and y-intercepts of *R(x)*.

X-INT: $y=0$ $2x^2-6x-7=0$

$x = \frac{-(-6)\pm\sqrt{(-6)^2-4(-7)(2)}}{2(2)} = \frac{6\pm\sqrt{36+56}}{4} = \frac{6\pm\sqrt{92}}{4} = \frac{6\pm 2\sqrt{23}}{4} = \frac{3\pm\sqrt{23}}{2}$

3.90 $\left(\frac{3+\sqrt{23}}{2}, 0\right)$ $\left(\frac{3-\sqrt{23}}{2}, 0\right)$ −0.90

Y INT: $x=0 \Rightarrow y = \frac{-7}{-8} = \frac{7}{8}$

$(0, 7/8)$

6

b) Write an equation for each vertical and horizontal asymptote.

VERTICAL ASYMPTOTES: $x=-2$, $x=4$

HORIZONTAL ASYMPTOTE: $y=2$ WHEN $x\to-\infty$, $x\to\infty$

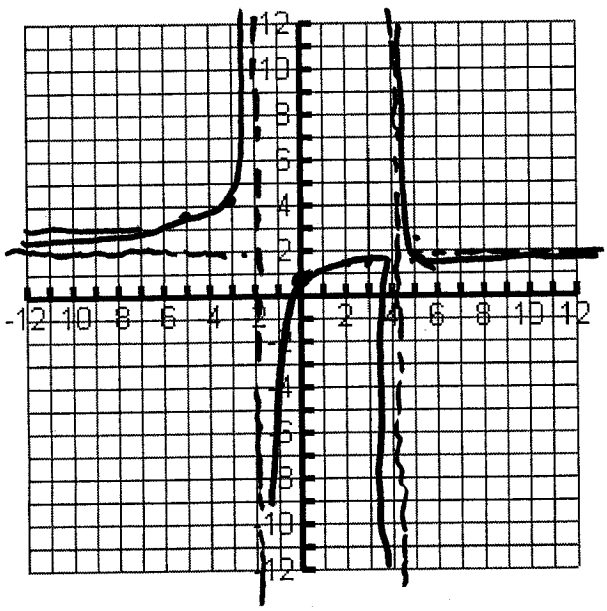
2

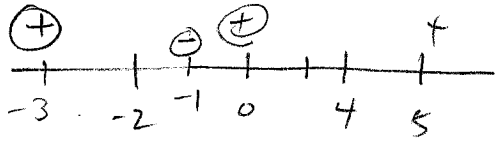
c) State the domain of *R(x)*.

DOMAIN: $\{x \mid x\in\mathbb{R}\text{'s} \text{ & } x\neq -2, 4\}$ $(-\infty,-2)\cup(-2,4)\cup(4,\infty)$

d) Graph R(x) by hand, using the above information and by plotting points. Make sure to include all information from above (intercepts, asymptotes, and proper end behavior).

8





(-3, 4.14)
(-1, -0.2)
(3, 1.4)
(2, 1.375)
(3.7, 1.11)
(4.2, 2.5)
(5, 1.86)

3. (9 pts) For each of the following rational functions, write an equation for every vertical, horizontal and/or slant asymptote. Only fill the boxes that apply, some boxes may remain empty!

1 EACH

	$h(x)=-\frac{3}{x}$	$h(x)=\frac{x+6}{x-2}$	$h(x)=\frac{x^2+3x+2}{x+3}$
Vertical	$x=0$	$x=2$	$x=-3$
Horizontal	$y=0$	$y=1$	
Slant			$y=x$

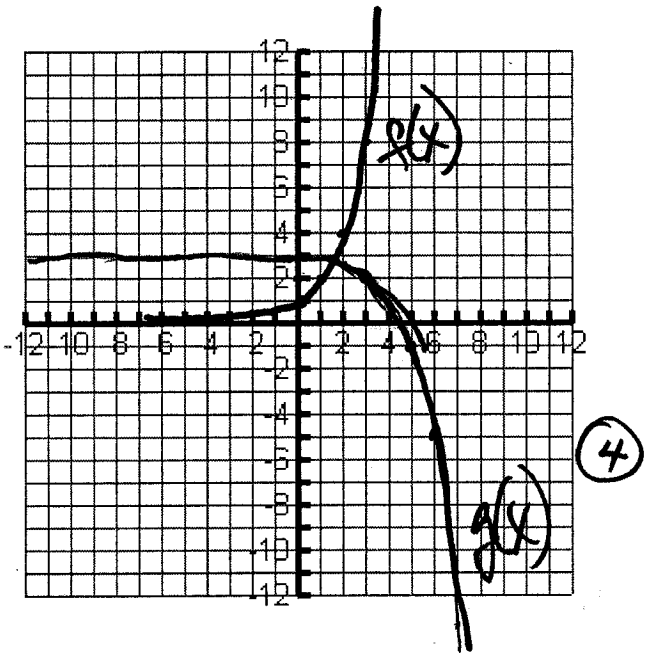
$-3\,|\,1\quad 3\quad 2$

$\quad\quad 0\quad -3\quad 0$

$\quad\quad 1\quad 0\quad 2$

$x+0+\frac{-2}{x+3}$

4. (8 pts) Below is the graph of $f(x)=2^x$. On the same set of axes, graph $g(x)=-2^{x-3}+3$ using transformations.



4

Describe the transformation in words.

SHIFTED RIGHT 3 UNITS, UP 3 UNITS & FLIPPED (UPSIDE DOWN) (4)

5. (9 pts) Simplify. Give an exact value for each. (NO DECIMAL approximations!!!)

a) $\ln\left(\frac{1}{e^3}\right)$

3 EXACT $\boxed{-3}$

b) $-3^{\log_8 512}$

$-3^3=\boxed{-27}$

c) $\log_8 6-\log_8 3+\log_8 2$

$\log_8\left(\frac{6}{3}\right)\cdot 2=\log_8 4$

$\Rightarrow 8^x=4$

$2^{3x}=2^2$

$\boxed{x=\frac{2}{3}}$

Formulas for this page: $A(t) = P\left(1+\frac{r}{n}\right)^{nt}$ $A(t) = Pe^{rt}$ $n(t) = n_0 e^{rt}$ $P(t) = P_0 e^{-kt}$

6. (8 pts) A sum of $5000 is invested at an interest rate of 8 ½ % per year, compounded quarterly.

a) Find the amount of the investment after 4 years.

(4)

$$A(4) = 5000\left(1+\frac{0.085}{4}\right)^{(4)(4)}$$

$$= 5000\,(1.02125)^{16} = \boxed{\$6999.76}$$

b) After what period of time will the investment total $12,000?

(4)

$$\frac{12000}{5000} = \frac{5000}{5000}\left(1+\frac{0.085}{4}\right)^{4t}$$

$$\frac{12}{5} = (1.02125)^{4t}$$

$$\ln\left(\frac{12}{5}\right) = 4t\,\ln(1.02125)$$

$$t = \frac{\ln(12/5)}{4\ln(1.02125)}$$

$$\boxed{t = 10.41 \text{ years}}$$

7. (12 pts) The half-life of palladium-110 is 4 days. After 20 days a sample has been reduced to a mass of 0.375 g.

a) What was the initial mass of the sample?

(4)

$$P(20) = 0.375 \qquad \frac{\ln(0.5)}{4}(20) = -5\ln 0.5$$

$$0.375 = P_0 e$$

$$\ln 0.375 = \ln P_0 + 5\ln 0.5$$

$$-5\ln 0.5$$

$$P_0 = \exp(\ln 0.375 - 5\ln 0.5) = \boxed{12g}$$

$$P(4) = \frac{1}{2}P_0$$

$$\frac{1}{2}P_0 = P_0 e^{-k(4)}$$

$$\frac{\ln 0.5}{-4} = \frac{-4k}{-4}$$

$$k = \frac{-\ln 0.5}{4}$$

b) Find a function that models the mass remaining after t days.

(2)

$$\boxed{P(t) = 12e^{-0.173287t}}$$

c) What is the mass after 3 days?

(3)

$$P(3) = 12\exp(-0.173287(3)) = 12\exp(-0.549860)$$

$$\boxed{P(3) = 7.13g}$$

d) After how many days will only 0.125 g remain?

(3)

$$\frac{0.125}{12} = \frac{12}{12}\exp(-0.173287t)$$

$$\frac{\ln\left(\frac{0.125}{12}\right)}{-0.173287} = \frac{-0.173287t}{-0.173287}$$

$$\boxed{t = 26.34 \text{ days}}$$

26 days 8 hrs 9.33 mins

8. (16 pts) Solve each equation for x. Give your answer correct to two decimal places.

a) $\log_x 343 = 3$

(4) $x^3 = 343$

$\boxed{x = 7}$

b) $\frac{5}{5}\ln(3-x) = \frac{4}{5}$

(4) $\ln(3-x) = \frac{4}{5}$

$3 - x = \exp\left(\frac{4}{5}\right) - 3$

$-3 \quad -x = 3 - \exp(4/5)$

$\boxed{x = 0.77}$

c) $10^{x+3} = 6^{2x}$

$(x+3)\ln 10 = 2x \ln 6$

(4) $x\ln 10 + 3\ln 10 = 2x\ln 6$

$-3\ln 10 \quad -3\ln 10$

$x\ln 10 - 2x\ln 6 = -3\ln 10$

$\frac{x(\ln 10 - 2\ln 6)}{\ln 10 - 2\ln 6} = \frac{-3\ln 10}{\ln 10 - 2\ln 6}$

$x \approx \boxed{5.39}$

d) $2^{x+4} = \left(\frac{1}{8}\right)^{-x}$

$2^{x+4} = 8^x$

$2^{x+4} = 2^{3x}$

$x+4 = 3x$

$-x \quad -x$

$4 = 2x$

$\boxed{x = 2.00}$

(4)

BONUS *(total of **10** extra points)*

ANSWERS MUST BE EXACT!!!!

A. Does $r(x) = \frac{2x+6}{x^2+4x+3}$ have a vertical asymptote at x = –3 ? **EXPLAIN why or why not.**

(5) $r(x) = \frac{2(x+3)}{(x+1)(x+3)} = \frac{2}{x+1}$

BUT $x = -3$ IS NOT IN THE DOMAIN BECAUSE OF ORIGINAL FCN.

$\Rightarrow$ NO VERTICAL ASYMPTOTE, BUT HOLE IN CURVE @ $x = -3$

B. If $2^x = 7$ what does $2^{-3x} = ?$

(5) $2^x = 7$ $\quad 2^{-3x} = (2^x)^{-3} = 7^{-3} = \boxed{\frac{1}{343}}$